

CHAPTER IX
SPACE AND TIME

WE will now return from the consideration of the mind's knowledge of itself and of other minds to its knowledge of the universe outside. We have traced the development of that knowledge from the private and intermittent phantasms of the solitary mind to the establishment of a public external world which is solid and independent, and which exists continuously even when no mind is aware of it. Fleeting private presentations have been replaced by permanent public objects. We had before us, at the close of Chapter VI, something like the ordinary world of everyday knowledge.

But to one important feature of that world we have given no consideration at all. Space and time, even as they appear in common knowledge—and taking no account of scientific concepts of them—do not belong to the world of the solitary mind. We have therefore to consider from what given elements in perception they have been developed, and what course that development has taken.

It must not, of course, be supposed that, because we have first dealt with the development of a public independent world, and only now come to the question of space and time,—it must not be supposed that such is the true order. A public external world is not first established and then afterwards space and time. Neither the logical nor the psychological order could be so represented without absurdity. Actually, no doubt, the two grow up together. Even logically the two developments are so closely interdependent that the separate consideration of them is only possible by means of an abstraction. But in matters so complicated we are compelled, for convenience of exposition, to consider first one branch of the development and then the other. And the order we have chosen to adopt is merely one of convenience.

The problems connected with the philosophy of space and time are so many and so difficult that anything like a

complete review of them is out of the question here. Such a review would require at least a separate volume to itself. I shall therefore confine my investigations to only one aspect of the question, that aspect which falls directly in the natural path of our inquiries. My object will be to show that our knowledge of space and time is riddled through and through with mental constructions; or, to put the same thing in a different way, that space and time as we know them are themselves mental constructions. And even my treatment of this one aspect of the problem can be no more than sketchy and diagrammatic, confining itself to the consideration of key positions. I shall detail some of what appear to be the main constructions involved. But I do not deny that there may be many more constructions in our knowledge of space and time besides those here to be laid down.

At once I shall be asked, 'what kind of space and time are you talking about?' It is now recognized that many kinds of space and space-time are possible, and that more than one kind may be actual in different parts of the universe. Is it Euclidean or non-Euclidean space or space-time that is to be the subject of the discussion? Is it the infinite space which was believed in by the older scientists? Or is it the finite space of Einstein or de Sitter?

I reply that the purpose of this chapter is not to consider advanced scientific ideas, whether of one school or another, whether those fashionable to-day or those which the fathers of our present scientists promulgated. Its purpose is to consider space and time as they appear in the common everyday knowledge of mankind. We have not as yet advanced far enough with our inquiries to begin thinking about science. We are at the very beginning. We have not yet justified even the commonest conceptions of space and time, conceptions such as the existence of empty space, the existence of three dimensions (not to speak of four as yet), the continuity and simultaneity of space, the identity of tactile with visual space, the existence of a common or public space and time. None of these ideas exist in the beginning for the solitary mind. We have to

begin at the beginning and develop them. That is the object of this chapter. Something will be said in later chapters of such questions as the Euclidean or non-Euclidean character of space, and the space-time of relativity mechanics. I will, however, say here in anticipation—since it will assist the reader to understand the general point of view adopted—that in my view all kinds of space and time, and of space-time *continua*, are mental constructions. They are alternative truths, of which the mind may incorporate into knowledge one or other according to its needs, and provided only that it neither contradicts the facts of perception nor the laws of logic.

For our starting-point we must go back again to the beginning, to the world of the solitary mind. Space and time as we now know them will not be found there. But something must be found there out of which space and time must have developed. This something will, of course, be an element of the given. To arrive at the starting-point for our present inquiry, therefore, we have to ask ourselves the question, out of what elements of the given have space and time been developed? What kind of rudimentary or embryonic space and time exist for the solitary mind? What is given to it as immediate and ultimate certitude? The answer to these questions has already been made fairly clear in a previous chapter. The solitary mind begins, not with any kind of space or time, but with extension-spread and duration-spread.

Our extension-spreads and duration-spreads are private to each of us. The red patch which I see has extension-spread. You see what we afterwards agree to believe is the 'same' red patch. But they are not given as the same. The sameness is, as we have abundantly shown, a later construction. And as the two red patches are not originally the same, so of course the extension-spreads which they carry are not the same. In the beginning therefore my red patch has one extension-spread and your red patch another. They are quite distinct from one another, and they exist in different private universes. It is, of course,

the same with duration-spreads. We start, then, not, as is sometimes said, with a multiplicity of private spaces and times, but with a multiplicity of private extension-spreads and duration-spreads.

The next point to note is that even within the private world of each solitary mind there is not a single continuous extension-spread and duration-spread, but a multitude of quite disconnected spreads. The red patch (which may be a pillar-box) disappears from my vision when I look away from it and reappears as a second and distinct red patch when I look again. Since the first red patch is not the same as the second red patch, therefore the extension-spread of the first is not the same as the extension-spread of the second. Nor are the two even continuous with one another. Other presentations with their spreads intervene. Or a period of unconsciousness may intervene. During intervals of sleep or unconsciousness of any sort the spreads disappear along with the presentations, and entirely new ones are created when consciousness again becomes active.

Further, we must not forget that the extension-spread which the solitary mind gets through sight is different from that which it gets through touch and muscular sense. The worlds of touch and sight are separate universes, and the spreads within them are numerically distinct and totally different in kind from one another.

Moreover, though the extension-spread of touch probably has three dimensions, the extension-spread of sight has only two. The visual world as given is flat and without depth. The third visual dimension is, as I shall endeavour to show in detail, a construction. This, of course, is a controversial question, and many philosophers will not agree with me. I shall give my reasons for my opinion later in this chapter.

Empty space, again, does not exist for the solitary mind at the beginning of knowledge. We cannot directly perceive emptiness, pure nothingness. Empty space, or at any rate empty visual space, is a construction. The given extension-spread does not continue beyond the edge of the presentation to which it pertains; or, if it does, it continues

only into another presentation. It does not continue into nothingness. Therefore empty space is not given.

For the same reason visual extension-spread does not continue outside the actual present visual field. It is bounded by the limits of the visual field. When I move my eyes round to a new set of objects, the first visual field not only disappears from view, but also, since its *esse* is *percipi*, ceases to exist. And its extension-spread ceases to exist along with it, and a new extension-spread comes into existence. For the solitary mind space does not spread out indefinitely into distance beyond what is actually seen. And certainly there is no room in such a mind for infinite space.

These, then, are some of the main characters of our starting-point. From this starting-point space and time as we now know them (whether Euclidean or non-Euclidean, whether finite or infinite) are developed by means of a series of constructions. And I shall now endeavour to set down a few of the main constructions in such order as may appear both logical and convenient. And in this endeavour I must once more guard against misconception. Not only are the following constructions probably not the only ones, but further it is not to be thought that the order in which I have placed them is a cast-iron order which is unalterable. Euclid adopted a certain order for his theorems, and it was a logical order in the sense that theorems which appeared earlier in his book are used as premisses for theorems which appear later. But it is well known that both the proofs and the order might have been different from what they are without detriment to the science. And if this is true of a complete and systematic development of the rigorous science of geometry, how much more will it be true of a series of theorems which profess to be no more than samples of the general procedure of the mind in the development of its knowledge of space and time.

First Construction.

That there exists for each mind a single continuous private visual space and a single continuous private tactile space.

For the solitary mind the extension-spread of a pre-

sentation ceases to exist along with the presentation as soon as the latter ceases to be perceived. That alone has extension-spread which is *now* present to the sight or touch. What lies outside the immediate visual and tactile fields has no existence, and therefore, of course, no extension-spread. Now let us, for the moment, use the word 'space' to mean the whole extension of whatever is comprised within a particular visual or tactile field. The single red patch has extension-spread. But the whole visual field which includes at one and the same time the red patch and a large number of other extended presentations we will call a visual space. Thus my space at this moment includes the extension-spreads of the presentations which represent the white piece of paper I am writing on, the brown patch of the table, a small piece of yellow wall, a row of books, &c. But the presentations which I perceived in the adjoining room before I came in here, and which I cannot perceive now, are in a different space which has now ceased to exist. Each mind, therefore, experiences a large number of spaces *which are not simultaneous but which follow each other successively in time.*

When a period of sleep or other interruption of awareness intervenes between the perceptions of two spaces, then those spaces are entirely cut off from one another. They do not in any way continue into one another. But when different spaces succeed each other continuously in time, as happens when I turn my head slowly round or when I pass from one room to another, then these successive spaces flow into one another. As I turn my head away from what I am now looking at, a new space begins to come into view which joins on to the old space. Suppose the first space is called *a, b, c, d*, and the second space *e, f, g, h*. While I am perceiving *a—d*, *e—h* is non-existent. For its *esse* is *percipi*, so that as it is not yet perceived it follows that it has not yet come into existence. When on the other hand I have turned my head right round and am perceiving *e—h*, then *a—d* is non-existent; for it has ceased to be perceived and has therefore gone out of existence. But when my head is turned only partly round, then I shall

perceive the intermediate space c, d, e, f , which comprises parts of both the previous spaces. It is in this way that successive spaces flow into one another.

But by the fourth construction of Chapter VI 'presentations may exist when no mind is aware of them'.

It follows that the extension-spreads of presentations—and, in general, that spaces—may continue to exist when no mind is aware of them.

By means of the fourth construction of Chapter VI the mind realizes the idea that presentations, instead of dropping out of existence when the mind ceases to perceive them, continue in permanent existence independently of the mind. As soon as the mind realizes this, the successive spaces of its experience become welded into one simultaneous space. For as the head is turned round, and as $e-h$ comes into view, it is no longer necessary to hold that $a-d$ has ceased to exist. It continues its existence unseen. So also before the head is turned it is not necessary to suppose that $e-h$ has not yet come into existence. It too has existed unseen. So that a, b, c, d, e, f, g, h now constitute a single simultaneous space the parts of which are perceived at successive times. For the solitary mind at its starting-point there existed many successive spaces. Now these are being replaced by one space the parts of which exist simultaneously but are perceived successively.

This process of unification need not stop anywhere short of completion. The successive spaces which I perceive during a continuous period of consciousness will obviously coalesce into one, since they flow directly into one another. But even the spaces which are separated by intervals of sleep or other forms of unawareness will follow the same rule. For general experience will teach us that unperceived spaces join on to perceived spaces. For example, I perceive to-day the space a, b, c, d while I am travelling in a railway train and looking out of the window. Then I doze for a moment, and on awaking I perceive the space i, j, k, l . These two spaces do not join on to one another, but as a result of the fourth construction of Chapter VI they are conceived as simultaneous and as

permanently existing. But the next day I pass the same way again and I perceive the space a, b, c, d and immediately adjoining it and continuous with it the space e, f, g, h . This is the space which I missed during my doze yesterday. By combining the two days' experience and holding fast to the view that all the spaces which I perceive continue existing even when they are not being perceived, I easily conclude that the space $a, b, c, d, e, f, g, h, i, j, k, l$ is one continuous and simultaneous space. In this way periods of unconsciousness are bridged over and all perceived and unperceived spaces coalesce into one.

Similar considerations hold of tactile spaces. Their continuity is learned by such elementary experiences as the passing of a finger along a surface. But these experiences are successive, and will give rise to a number of successive spaces. The permanent existence and simultaneity of these tactile spaces is a deduction from the fourth construction of Chapter VI, similar in all ways to the deduction just made in the case of visual space.

The curious opinion of Platner and others that men born blind could not obtain from touch alone any idea of space, but that for them time must serve instead of space, is now known to be erroneous.¹ But it is worth noting that it was apparently based upon a failure to observe that the perceptions of visual and of tactile space *as given* are precisely on a par as regards successiveness. It was pointed out by those who held this opinion that the sensations which arise from such experiences as passing one's finger along an edge or a surface are all successive. They would easily give rise, therefore, to the idea of time, but could not originate the notion of the simultaneously co-existing parts external to one another which is of very essence of space. It was evidently thought that sight has some advantage over touch in this respect. It was thought that though we only touch things successively we see them simultaneously. It was thought that the panorama of visual space lies before us and that the parts of it are *seen*

¹ See *The World of the Blind*, by Pierre Villey, p. 197 (English translation).

as simultaneous. But this is manifestly a mistake. The visual field is exceedingly small. What lies within one and the same visual field is no doubt seen as simultaneous. But the spaces (and their contents) which are so large that they cannot be got within one visual field can only be seen by successive acts of vision. The sweeping of the eye round the four walls of the room, or the action of the person in a moving train who looks out of the window on the passing scenery, is precisely analogous to the passing of the finger along a surface. No doubt vision gives a *small* simultaneous space, namely that which can be got into one visual field. But touch also gives as simultaneous the space which can be felt by one stretch of the hand or other part of the bodily surface. The hand's breadth or other such tactile unit corresponds to the visual field. No doubt it is true that vision has certain advantages over touch. But our point is that neither gives a single space. Both give a series of successive spaces. And the one single space, whether tactile or visual, is built up by the mind out of these. In spite of certain obvious advantages which sight has over touch, there is no difference of principle between the construction of visual space and the construction of tactile space. Both begin with a series of small successive spaces. And both convert these by means of a mental construction into a single simultaneous space.

Thus there comes to exist for each of us a single private visual space and a single private tactile space. That is as far as the present construction professes to take us. And we will now examine the characters of the construction.

The essential features of space with which we are concerned are (1) continuity, and (2) simultaneity. Continuity as here understood does not refer to the well-known mathematical conception which goes by that name, but has reference only to the common fact that every part of space flows without a break into the next part. Space and time are both continuous, but space alone is simultaneous, i.e. its parts are all simultaneously existent. This simultaneity is what distinguishes space from time.

Now the character of continuity is *given*. The imme-

diated visual field consists of parts which are continuous, and the space within the visual field is therefore given as continuous. Moreover its continuity with the next adjoining visual field, which arises when one turns one's head or walks into the next room, is also given. It is true that there exist breaks in the given continuity at points where sleep or unconsciousness of the external world supervene. These gaps are filled up by the mental construction.

Simultaneity, except within the very narrow limits of the visual or tactile field, is not given. The simultaneity of the paper and the table on which I am writing is given, since both are actually perceived in the same visual field at the same time. But the simultaneity of the space I now see with the space which I see a second later when I turn my head round is not given, since the two spaces are seen successively. In this case their continuity is given, but not their simultaneity. Nor is the simultaneity of two spaces outside the limits of the perceptual field an inference from anything which is given. While I am now perceiving the space $a-d$, I have no reason whatever for believing that the space $e-h$ is in existence. It cannot be inferred from anything which I am now seeing. The simultaneity of the two spaces, therefore, cannot be perceived, since they are perceived successively. And it cannot be inferred, since no inference can possibly pass to an unperceived existence. The unperceived space here is in exactly the same position as the red patch or other presentation which is unperceived. We saw in Chapter VI that the mind can have absolutely no reason for believing in the existence of an unperceived presentation, i.e. its existence cannot be inferred from anything that is perceived. And it was for this reason that we concluded that the belief is a construction. In exactly the same way the mind cannot have any reason for believing in the existence of an unperceived space, i.e. of a space now lying outside the visual or tactile field. The existence of such a space can neither be perceived nor inferred. Therefore it must be a construction.

Without any construction we might indeed have a continuous series of successive spaces—so long at least as we

do not fall asleep. But the idea of a single space, all parts of which are simultaneous, cannot be reached without construction.

We have held this idea to be a construction because it can neither be perceived nor inferred. It may be pointed out, of course, that it is an inference from the fourth construction of Chapter VI. That construction established that presentations may exist outside the present perceptual field. That of course implied that the space or extension-spread of the presentations must similarly exist unperceived. And the present construction of a single simultaneous space merely consists in drawing that deduction and combining it with the given experience of continuity.

This is quite true. But the fact that our belief in such a space is in this way an inference from a previous construction does not nullify the view that this belief is itself a genuine construction. The essential character of a construction is that it is a belief in something which is not given and cannot be inferred *from what is given*. It may, however, be inferred from another construction. Constructions are in fact frequently connected to one another by links of logical implication; so that we get chains or systems of constructions. Scientific knowledge consists largely, not of isolated constructions, but of networks of them, each construction being logically implied by all the others. It is in this way that knowledge becomes systematic. But so long as none of the constructions which are members of the system can be deduced logically from any perceived fact the whole system will be constructional, not factual. So it is in the present case. The present construction regarding space is practically an inference from the previous constructions regarding presentations. But it does not for that reason cease to be a genuine construction.

This construction is existential in type. One might be at first disposed to class it as unificatory. For it seems to reduce the many spaces to one. But this would be a mistake. In the true unificatory construction two or more

different things are declared to be the 'same', i.e. to be numerically identical. For example your red patch is declared to be the same as my red patch, although there are palpably two, not one, in factual existence. But in the construction which we have just described as regards space no such process of declaring two different things to be numerically identical takes place. The many private spaces which are welded into one are not declared to be identical but merely to adjoin each other and to flow into each other. The space $a-d$ is conceived as running into the adjoining space $e-h$. They are also conceived as simultaneous. But no one supposes that they are identical. Their identity would, however, be implied if the construction were unificatory.

The essence of the present construction is that it posits the existence of unperceived spaces simultaneously with perceived spaces. It constructs, therefore, an unperceived existence, and is accordingly an existential construction.

It possesses accordingly the characters of all existential constructions as laid down in Chapter VI. It can only be accurately expressed in the form of an hypothetical judgment having an impossible antecedent clause. It means 'If I were now somewhere else as well as where I actually am, I should perceive another space simultaneously with the space I am now perceiving, and continuous with it'. The condition is impossible because I cannot be in two places at the same time.

Second Construction.

That there exists for each mind a single continuous private time.

This construction does for time what the last construction did for space. The two constructions are parallel, and the present one may be very shortly treated. The many spaces were welded into one space of which all the parts were (1) continuous, and (2) simultaneous. Of these two characters only the first applies to time. Simultaneity is absent from it. Hence all that the present construction has to do is to weld the many times into one continuous

time. While I am awake and aware, I perceive a continuous stream of presentations in a continuous time. The continuity is here given in immediate perception. But suppose I go to sleep for half an hour. When I wake up a new time begins which is not continuous with the last. It is only in this way that there exist for each mind at the starting-point of its knowledge a number of disconnected times.

The construction which unites these is very simple and obvious. Like the last construction this one also is involved in the fourth construction of Chapter VI. Presentations exist when I am not aware of them. I learn from other minds that while I was asleep the red patch which I saw before I fell asleep continued in existence until I saw it again on awakening. It follows that the two disconnected times of which I was aware were joined by a stretch of time which was unperceived by me.

The construction is existential, since it invents an unperceived time. It means 'If I were awake while I am asleep, I should perceive the flow of time'.

Third Construction.

That visual space and tactile space (and any other perceptual spaces which may exist) are identical with one another.

The words contained within brackets are not important. It might be asserted that in addition to visual and tactile spaces there exist auditory, olfactory, or even gustatory spaces. If any such spaces exist—regarding which I express no opinion—they will at any rate come to coalesce with the visual and tactile spaces by means of constructions essentially similar in principle to that by which the visual and tactile spaces coalesce. The words within brackets are inserted merely for the sake of completeness and to cover that possible point. And I shall not refer to the question again, but shall confine my inquiries to the spaces which are admittedly the only really important ones, namely visual and tactile spaces.

The present construction is merely a corollary of the sixth construction of Chapter VI, which asserted 'that with the different senses we perceive the "same" objects, and

that the worlds of the different senses are in general identical with one another'.

We saw the difficulty of identifying the visual table with the tactile table. They bear no resemblance to one another at all. Yet the mind identifies them. It must clearly be the same with the spaces which they occupy. Visual extension-spread does not bear any resemblance to tactile or muscular extension-spread. They are totally incomparable, incommensurable. But the two are associated in experience in such a way that the one is always a sign of the other. When the construction of the 'thing' underlying appearances or presentations comes before us, as it has already in Chapter VI, then those characters of the 'thing' which are in different sense-worlds, but are signs of one another, become identified. The solid appearance to the eye is identified with the sense of resistance to touch. And similarly the visual extension-spread is identified with the tactile extension-spread. They are but the two appearances in the two sense-worlds of the 'same' space which is occupied by the 'same' object or 'thing'.

The present construction can be exhibited as a direct deduction from the sixth construction of Chapter VI. For if the tactile table is identical with the visual table, then the spaces which they occupy must be the same.

This construction, like the last, is an inference from a previous construction. But it is not an inference from any perceived fact. It is possible to prove it in the sense that it can be deduced from a previous judgement. But that previous judgement is itself a construction, an assumption which cannot be proved. Therefore the present construction cannot be proved, and is in the last resort merely an assumption which fits in with other assumptions. It is therefore a genuine construction.

Its character follows that of the sixth construction of Chapter VI. It is unificatory.

Fourth Construction.

That there is but one space common and public to all minds, and one time common and public to all minds.

By this construction private spaces and times disappear and cease to exist. They are all swallowed up in a common space and a common time. So far we have of course been dealing only with private spaces and times. The many successive spaces of each mind fused into one. Then the private tactile and the private visual space of each mind fused into one space. Now the final step in the same process is taken when the many private spaces coalesce into a single public space, and the many private times into a single public time.

It seems hardly necessary to labour the derivation of the construction after the detailed description we have given of previous similar constructions. As with the last two constructions, the present one follows closely the development of the public external world given in Chapter VI. The many private worlds having coalesced into one public world, it is obvious that the many private spaces and times will coalesce into one public space and one public time. The present conclusion is thus simply a corollary to the second construction of Chapter VI, which laid it down that 'the corresponding presentations of different minds are identical, and that there are not many universes, but only one'. If the red patch which I see is identical with the red patch which you see, then it follows that the spaces and times which they occupy must be identical. This at once yields the present construction.

The construction is of the unificatory type. And it is a genuine construction as being incapable of proof, except by deduction from a previous construction.

The order of the constructions of this chapter up to date is no doubt open to comment. By the third construction my visual space was identified with my tactile space, so that the various private spaces of each separate mind are reduced to a single all-embracing space, which is, however, still private to that mind. By the present construction my private space is declared identical with yours and with the private spaces of all other minds, so that the many private spaces are reduced to one common space. It is possible that this order of constructions might have been different.

For example, we might have exhibited the mind as first identifying the various private visual spaces with one another to make a public visual space; next as having done the same thing with the many private tactile spaces; and finally as having identified the public visual space with the public tactile space so as to produce the single space, common to all the senses and to all minds, which is the space of our ordinary present-day knowledge.

There would be much to be said for such an order. And this illustrates how, as I have already pointed out at the beginning of this chapter, different orders among the various constructions of the mind on all subjects may be possible. Thus the order adopted in the present chapter does not profess to be rigidly exclusive of all other orders, and has been chosen largely from considerations of convenience.

Fifth Construction.

That visual space possesses a third dimension.

Opinions differ as to the origin of the third or depth dimension in visual space. It may be held to be either (1) given, or (2) inferred from what is given, or (3) constructed.

The view (1) that it is given, is apparently that taken by William James and by more recent writers such as Dr. C. D. Broad. The view (2) that it is inferred, is that of Berkeley. The view (3) that it is a construction, is the opinion which will be adopted in this book. For the existence of the third visual dimension cannot be proved by an inference as Berkeley thought.

It will be observed that our statement of the construction in the heading refers to visual space and leaves out all reference to tactile space. That is because, in my view, tactile space stands on a wholly different footing from visual space in this respect. I believe that the third dimension of tactile space is given in perception, is there from the start; but that in visual space only two dimensions are given, so that the third has to be constructed.

If we take touch and muscular sense together, the space

which they yield appears to be three-dimensional from the outset. The three dimensions are immediately given. They are not, however, given by pure touch alone without the aid of muscular sense. If I stand in the garden by the outside corner of my house, I can pass my finger-tips along the wall which faces me, and when they reach the corner I can then pass them along the wall which lies at right angles to this, i.e. the wall which goes away from me into the depth dimension. But as far as pure touch is concerned there will be no change of sensation after passing the corner. I shall receive three successive sensations, namely (1) the feel of my fingers passing along the wall which faces me—which may reasonably be supposed to give rise to the idea of a surface, (2) the sharp feel of the corner, and (3) the return of the feeling of my fingers moving along a surface. The third sensation will be identical with the first, though it will be divided from it in time by the second. But the interruption of the corner will make no difference. And the third sensation being identical with the first, there is nothing in the experience which could give rise to the idea of a change of direction or dimension.

The two surfaces at right angles thus give rise to identical tactile sensations. The muscular sensations, however, will differ. While I am passing my hand along the wall which faces me, my arm is being moved from side to side. After it passes the corner my arm must be stretched out away from me to follow the wall. The muscular sensations of the two movements will be quite different from one another, and this difference will be sufficient to give rise to the idea of a dimension different from the two surface dimensions, i.e. to the third dimension.

And it would seem that by feeling the angle of a cube I should be able to receive a direct impression of three dimensions. I pass my finger from the point along the three converging edges. The pure touch sensations in the pulp of the finger will not give dimensions. They will only give line or surface. But the muscular sensations of hand and arm will differ for each of the three edges and will constitute rudimentary perceptions of the three dimensions.

It is for this reason that I hold that in tactile-muscular space (which is what is ordinarily called tactile space) the three dimensions are *given*.

But visual space is totally different. It is flat. To the extent of holding that the depth dimension is not given, i.e. not directly perceived, I agree with Berkeley. He long ago propounded the view that 'distance, of itself and immediately, cannot be seen. For, distance being a line directed endwise to the eye, it projects only one point in the fund of the eye, which point remains invariably the same, whether the distance be longer or shorter.'¹ Our estimates of the distances of objects, he explained, are inferences. In the case of objects close at hand there are the muscular sensations involved in the convergence of the two eyes and the accommodation of the eye muscles. In the case of far away objects we judge by the apparent size of the object, by our estimate of the sizes of the objects which intervene, by the colour ('blue hills far away'), and a number of other factors.

Berkeley's opinion held the field, and was almost universally regarded as established, until quite recently. It is now widely disputed. William James considered that the depth dimension is as much sensationally original as the two superficial dimensions. He thought that what we primarily experience is a sensation of *volume*, and in proof of this he appeals to many experienced feelings, such as that glowing bodies appear 'roomy', look luminous 'through and through', and so forth.² But these experiences prove nothing, except what is already admitted, namely that *now*, after our sight has been developed and educated, we *feel* that we have perceptions of distance and depth. The question is whether, when we feel that we can immediately sense distances and depths, we have before us real cases of pure perception, or whether they are not rather cases in which cognitive processes of construction or inference have been involved, but have sunk, through use and wont, down to the perceptual level and become

¹ Berkeley, *A New Theory of Vision*, paragraph 2.

² James, *Principles of Psychology*, Chapter XX.

embedded there. Suppose I judge that an object is at a certain distance from me because I see certain other things intervening which I know by experience to be of such and such a size. A great deal of thinking is involved here. Yet such judgements become automatic by frequent practice. They become unconscious, and thereby sink to the level of the immediate. They are then mistaken for perceptions. Now all that James does is to quote numerous examples of experiences of what *appear* to us to be direct perceptions of the voluminousness of objects. But this procedure simply begs the question. It does not prove that the supposed perceptions are really such, it does not prove that they are aboriginal or known to the solitary mind. My view is that they are a product, not an original element. They are the result of deep unconscious constructions, of cognitions the separate discursive steps of which have sunk into the subconscious, or perhaps have never risen out of it. Such unconscious thought-processes appear in the upper levels of consciousness in the guise of immediate feeling or 'intuition'. These feelings are then seized upon by writers like William James to prove that depth and distance are aboriginal perceptions!

Dr. C. D. Broad is one of the more recent writers who appear to hold views essentially similar to those of James. Dr. Broad says, 'it is perfectly obvious to me that I do sense different patches of colour at different visual distances'.¹ This makes it clear how each of us is, after all, enclosed in his own private world. For Dr. Broad can see distance. I, unfortunately, cannot. It is 'perfectly obvious' to me that I do *not* sense patches of colour at different visual distances. In the garden in front of my window in this tropical country in which I am writing is a coco-nut palm, and at some distance behind it another coco-nut palm. I can see that the trunk of the former appears larger than that of the latter, and that the details of the fronds and the lines of the bark are on a larger scale. I see various other differences between the appearances of the two trees. And I see the grassy ground which intervenes

¹ C. D. Broad, *Scientific Thought*, p. 295.

between them. Experience has taught me to judge from all these facts that one lies at a distance behind the other. But I am absolutely unable on analysis to detect in myself any direct perception of visual depth.

If there is such a direct perception of visual distance, how is it that all the stars appear to be at the same distance from us? We know that enormous differences of distance exist, that one star is a hundred times further from us than another. How is it that we have not even the faintest trace of a perception of this? That we could not measure the exact relative distances by eye might be understandable. But surely there ought to exist *some* difference between the perception of the star which is three light-years away and the perception of the one which is three hundred light-years away. There is in fact no difference at all, and the reasonable way of explaining this is to think that we have no perception of distance. Will it be replied that we have the sense of depth for objects which are close to us on the earth, but that it fails us when we come to deal with interstellar distances? But this is purely arbitrary, an *ad hoc* assumption invented to explain away the difficulty.

Again, if there is a direct sense of visual distance, how is it that we are often deceived about such matters? Flat pictures can be made which in suitable circumstances will actually be taken by an unsophisticated observer to be a group of objects having depth and three dimensions. A flat light-coloured disk can be painted and shaded in such a way that it will be mistaken for a sphere. And such a flat disk and a real sphere placed side by side may be indistinguishable from one another. This shows that the sensations received from a three-dimensional world are identical with those received from a two-dimensional world, and that there is no special sensation of distance peculiar to the former. It shows that there is no difference between the *perceptions* of three-dimensional and two-dimensional objects, but that the difference lies in the *interpretations* which our intellects place on them. And it surely follows from this that no perception of distance exists. And if we are deceived by a flat picture which

looks as if it had depth, this impression of depth must be a part of our interpretation, since sensation as such never deceives us.

I therefore think that Berkeley was right, and that all that is given in sight is a flat, coloured, two-dimensional surface.

The third dimension not being given, we have next to ask whether it is inferred from anything that is given. Now it is true that, the general idea of visual distance having once obtained an entrance into the mind, the judgement of particular distances may be a matter of inference. I know that certain peculiarities of sensation (such as the larger or smaller appearance of an object, the appearance of intervening objects, the muscular sensation caused by the convergence of the eyes on a very near object) are signs of greater or lesser distance from the eye. But all these inferences presuppose that visual space has three dimensions and that the mind is aware of this. They presuppose that the idea is already in the mind. The fact that one coco-nut palm looks twice as big as another does not prove that the first is nearer to us. From this difference in size you could not infer the existence of a depth dimension. For the same appearance, the same difference, exists in the painted picture of the coco-nut trees, and in that case there is no depth dimension. But, when once it is known and admitted that three dimensions exist, then various phenomena of visual sensation become signs from which the greater or less distance of objects may be inferred. So that, although particular distances may be inferred in the various ways well known since the time of Berkeley, the existence of visual depth is not itself an inference from anything given in vision. There is clearly nothing in the fleeting colour patches which we see (and we see nothing else) from which the idea of depth could be deduced.

Since the third visual dimension is neither given nor inferred from what is given, we are left with the alternative that it must be a mental construction. The only question which we have to face is how and why the mind made this construction. Let us turn to that issue.

Suppose that we perform the experiment, beloved by all philosophers, of rotating a penny in such a way that, having first appeared circular, it becomes an ellipse of increasing eccentricity, until it at last (when placed edge-wise on) shows only as a very narrow rectangle or band. Now, if we had only the sense of sight and no sense of touch, we should probably explain this series of appearances as mere *change of state*. I need hardly remind the reader that change is, as far as outward appearances go, of two kinds. Change of spatial position is called motion. But there are many changes which are not motions, as when the leaf changes its colour from green to yellow. This is change of state. Now my point is that although *we* explain the changes in the appearance of the penny as due to motion (rotation), this explanation would not occur to a being possessed of no sense except that of sight. He would not have attained the idea of a third dimension through touch, and he could not attain it through sight. He would therefore be without it. Nor would the idea dawn upon him as a result of watching the rotating penny. For he would explain that quite simply to himself, not as due to motion at all, but to change of state. He would say that the circular brown patch had contracted to an ellipse and then to a narrow rectangle. This only involves change of colour. For suppose the background on which the penny appears is white. Then the change from circle to ellipse and ellipse to rectangle is nothing more than the fact that a part of the flat surface of the visual field which was brown becomes white. Such a being, then, will not explain the phenomena of the rotating penny by means of the third dimension.

But now suppose that I have the sense of touch, and that the rotation of the penny has been caused in the usual way by my turning it round in my fingers. While it appeared circular I could pass my finger across the flat disk. From this I received certain tactile sensations. Now it is part of the constructed concept of external existence—the construction of which the mind has already completed—that the object of touch is identical with the object of sight.

I have therefore identified the tactile extension-spread given in the act of passing my finger across the penny with the visual extension-spread of the penny. Although they bear no resemblance to one another at all, and although they exist in quite separate universes, I have yet for my own purposes constructed the view which consists in ignoring these differences and asserting that they are the 'same'.

I now turn the penny edgewise on. The circular visual extension-spread has *disappeared*. But I can still pass my finger across the surface of the penny even though I cannot see that surface. The tactile extension-spread therefore is still perceived and still exists. Here is a contradiction. The visual and the tactile extension-spreads are supposed to be identical, yet one has gone out of existence while the other still exists. In this dilemma I must do one of two things. I must either abandon the belief that the tactile and visual extension-spreads are identical; or I must assume that the visual extension-spread still goes on existing when it is unperceived. If I adopt the former alternative, my entire world of public external existence, so elaborately built up, comes tumbling down, cracks to pieces. So I am compelled, in order to be consistent, to assume that the visual extension-spread goes on existing when it is not seen. This idea fits in very well with my general scheme of things, in which I have assumed the continued existence of things when they are not being perceived. So I adopt it without hesitation.

But the idea of a flat surface which is out of sight is precisely the concept of the third visual dimension. This is not, of course, a definition of the third dimension which would satisfy a geometer. But geometry with its straight lines and right angles is a later construction which does not exist at the primitive stage of the development of knowledge which we are discussing. The conception at which we have arrived is the only possible definition of the third visual dimension in terms of pure perception. Depth or distance is simply a line or surface in space which is so placed that we cannot see it.

In this way, then, the third visual dimension is constructed. That it is a genuine construction is shown by the fact that it can neither be perceived nor can its existence be proved. That it cannot be perceived is the discovery which we owe to Berkeley. That it cannot be proved follows from first principles. For it is by hypothesis a visual appearance which is unseen. It is the old story of the impossibility of proving the existence of a percept when no one is perceiving it. We can no more prove the existence of the third visual dimension than we can prove that the table continues existing in the night when no one is there to perceive it.

It may, of course, be represented as an inference from previous constructions. It follows, in fact, from the view that tactile and visual space are identical. For we know by perception that tactile space has three dimensions. And therefore, if visual space is identical with it, visual space too must have three dimensions. That is really the pith of the present construction. But the view that tactile space is identical with visual space is itself a construction, an unprovable assumption. Therefore in the end the existence of a third dimension in visual space is also a construction, an unprovable assumption.

It is a construction of the existential type. It is therefore correctly expressed only in the form of a hypothetical proposition with an impossible antecedent. The assertion that there is a visual extension of the penny along an invisible line can only mean 'If I were in a position which I am not in (i.e. rotated through a quarter of a circle) I should see the extension-spread of the penny'. This is an impossible condition since I cannot both be in my present position and in another position at one and the same time.

Sixth Construction.

That empty visual space exists.

That empty tactile space exists is apparently given. Resistance to the pressure of one's hands gives the material object or filled space. The absence of resistance with consequent free muscular movement would seem to give the

rudiments of an empty tactile space. I pass my hand along a surface. I receive (1) tactile sensations in the pulp of the fingers, and (2) muscular sensations in the arms and hands. The combination of these two gives filled space, i.e. the existence of objects (presentations) with their extension-spread. The occurrence of the muscular sensations without the corresponding tactile sensations, as when we make the same movements as if we were passing the hands along a surface without actually touching a surface, would seem to give the necessary foundation for the idea of empty tactile space.

But since visual space is as perceived wholly different from, and having nothing in common with, tactile space, we cannot therefore conclude that what is true of the latter is also true of the former. And when we examine visual space it is clear to us that empty visual space is not given. You cannot *see* empty space. What you see is always a coloured surface, and the coloured surface has a flat extension-spread. The whole of the visual field is full. It is a full space of two dimensions. The idea of a visible emptiness or a visible nothingness is something which the mind is unable to picture. It is unthinkable.

Full space is thus given. It is just as much a sensed quality of the visual field as colour. It is, in the form of extension-spread, primarily simply a quality of objects. And it would appear to be a somewhat extraordinary circumstance that the mind should suppose that the quality of an object can extend beyond the object itself and exist on its own account without the object. Yet this abstraction is exactly what we mean by empty space. We *now* think of space and matter as distinct from one another. We speak of matter as 'occupying' space, and of space as something existing in its own right and 'containing' objects. But originally we must surely believe that matter and space were inseparable, and that the hypostatization of space as itself a sort of 'thing' which can exist without matter (empty space) is a construction of the mind.

How and why has this construction come into being? One common explanation is that we are compelled to

assume empty space in order to allow for motion. But a very little reflection will show that this is not the case. We must remember that we are speaking here both of visual space and visual motion. Tactile motion is, of course, a totally different thing from visual motion, until the two are artificially identified as part of the general construction which identifies the visual object with the tactile object. Now there is no justification for the view that we are compelled to assume the existence of empty visual space in order to explain the existence of visual motion. For visual motion is much more naturally and simply explained as mere change of state, in fact as change of colour.

This will perhaps be obvious from what we said regarding the rotating penny under the head of the last construction. But let us take also another example. Suppose that I am looking vertically downwards from the ceiling on to a billiard table below me, and that a white ball is rolled across it. What I actually see at any moment, i.e. what is given, is a green surface with a small white disk on it. The white disk is seen to pass from one side to the other. The solitary mind, aware of nothing but the flat colour patches, not aware of either the existence of solid objects or of the third visual dimension, certainly would not explain the appearance of the green surface with the moving white disk as due to the motion of anything through empty space. It could quite easily explain the phenomena by supposing that successive portions of the green cloth (or green surface presentation) *turn* white, and that the portion where the white patch was a moment ago *turns* green again. This way of regarding the matter will satisfactorily explain all lateral motion across the field of vision. As to motion which we are now accustomed to explain as motion along the line of vision, this only *appears* as an increase in size (if the object is approaching) or a decrease in size (if the object is receding). The white disk on the green background grows larger or smaller, takes up more or less of the visual field. The white swallowing up the green, or vice versa, can obviously be far more simply explained as

change of colour than by elaborate constructions of third dimensions and empty spaces.

To have explained all visual motion as mere change of colour in the visual field would be just as legitimate and just as 'true' as to explain it by the hypothesis of free movement in empty space. The mind might just as well have adopted the former explanation, built it into the system of knowledge, and constructed the rest of its world in accordance therewith. This would have been an alternative truth, and would no doubt have so altered the whole of our outlook on the external world that a whole system of constructions, a whole system of knowledge different from that which we now possess, would have grown up. We should have had an example, not only of a single alternative truth, but of a complete alternative system of knowledge.

Why, then, has the mind not done this? Why has it adopted the more complicated method of explaining the experienced facts by the hypothesis of free movements in empty space? The ultimate answer is that to have adopted the view that the facts are due to mere change of colour would have come into conflict with the already constructed concepts of external existence and the equivalence of the senses. Those concepts had already been constructed by the mind before it came to deal with the problem of visual motion. And the mind could not be expected to go back on its tracks, to throw overboard its public external world, and to begin its world-building over again on another plan. Of the two alternative explanations it will choose that which is consistent with its previous constructions. Let us see how this is.

By the fifth construction of this chapter the mind constructed the third visual dimension. When once this has been done, the concept of empty visual space follows as a necessity. Two green boxes stand against the white wall of my room. I can see the wall between them. If there were no third dimension, I should say that what I see is a flat continuous coloured surface with a white patch in the middle and green patches on each side. But now, through the concept of the depth dimension, I learn that the white

wall is not really between the two boxes, as it appears, but behind them in a different plane. I believe this because, by passing my hand along the sides of the boxes which lie out of sight along the depth dimension, I feel that they possess a third tactile dimension, and my constructed belief in the equivalence of sight and touch compels me to think that there must also be an invisible visual dimension. Thus I am forced to the conclusion that the white patch (the wall) is *not* between the two green patches, but behind them. What, then, *is* between the green patches? If it is not the white wall which I *see* between them, then it must be nothing. There is clearly an extension-spread between them, a distance, which I had supposed to be white. I had supposed that colour and extension-spread always went together; that a colour must always be extended, and that extension must always be coloured. I am now compelled to believe that there must exist an extension-spread between the two boxes which has no colour and no other quality of any sort, or in other words that extension-spread must exist on its own account by itself. When I arrive at this conclusion I have constructed the concept of empty visual space.

Thus we see that in the last resort empty visual space is a concept which is forced upon us by our original construction of a public external world. In order to arrive at that world we had to identify the world of touch and the world of sight. That was part of the construction. We now find that that identification compels us to construct empty visual space. Hence it is now clear that to explain the visual phenomena of movement as mere change of colour, and to deny the existence of empty visual space, would have been inconsistent with our previous construction of a public external world. Therefore we adopt the belief in the existence of empty visual space.

Empty visual space is therefore itself a construction which is inferred from previous constructions. It is not given. And it is not inferred from anything that is given. It is constructed for the purpose of squaring with conceptions already constructed. We could certainly have built

up our world without it. But if so we should have to have built it up on some totally different plan. And it is possible that we should have had to give up any active world-building at all and to remain content with our separate worlds of private phantasms.

Seventh Construction.

That there exist relations of equality between distances, areas, and volumes in space; and that exact measurement becomes thereby possible.

This is the construction which makes geometry and the application of mathematics to space possible.

The special point which it is necessary to make clear here is this: that when we say that any spatial magnitudes, say two straight lines AB and CD , are equal, we are stating something which is not given and cannot be proved. *It is impossible, in spite of Euclid or any other geometry, to prove by any valid reasoning that two straight lines, or two areas, or two volumes, are equal to one another.* And when we speak of them as equal, we are making use of a conception of space which is a pure construction, a fiction invented to suit the mind's purposes.

All measurements of space, it will be obvious, are based upon the concept of equality, and would be impossible without it. When I say that a rod is six feet long, the meaning of this assertion is that if it is divided into six *equal* parts, each of these parts will be *equal* in length to the standard foot. Measurement is only the application of the concept of equality to particular lengths, areas, or volumes in space.

The fundamental character of measurement in modern science will also be obvious. It is even said that exact physical science is concerned with measurement and with nothing else. Such science, Professor Eddington has impressed upon us, is purely a matter of pointer readings. But the pointer readings on a dial, whether they measure electrical charges, temperature, the pressure of light, the frequency of vibrations, or whatever else, are dependent upon the supposed fact that the dial is divided into *equal*

spatial segments. Thus if physics is based upon measurement, and measurement is based upon the concept of equality, then if equality is a mental construction, it will not be too much to say that practically the whole of physics is a system of mental constructions. The far-reaching character of the present section will therefore be clear.

Let us see, in the first place, what immediate perception gives us in the way of material for the concept of equality. It is clear that the relations of larger and smaller, and of rough similarity in size, are given. When a man stands beside his house, the fact that the house is bigger than the man is immediately perceived by the eye. When two walking sticks are placed upright on end we immediately perceive the fact that their tops are level, and that they are therefore similar in length. And it might well be supposed that these facts are sufficient to give us the perceptual basis of exact measurement, and that they disprove our statement that the concepts of equality and measurement are constructions.

But this is not so. Let us compare spatial magnitude with intensive magnitude. It is given in perception, for example, that some pains are more intense than others, and that some are roughly similar in intensity. But it is nevertheless not possible to measure the intensity of a pain, or to say that one pain is twice as great as another. The same is notoriously true of all psychic elements. I know that I was more angry when you slapped me in the face than when you merely turned your back on me. But I cannot measure the intensity of my anger in either case. It is true that attempts are made, with some success apparently, to measure the strength of the bodily commotions which accompany the emotion. But that is not the same thing as measuring the intensity of the psychic experience. It is well known that the difficulty of exact measurement in psychology is one of the principal reasons why psychology falls short of being an exact science like physics.

This shows that the mere fact that the relations of greater and smaller, and of similarity in size, are given in perception, is not in itself a sufficient basis for measure-

ments of space. It shows that the fact is not inconsistent with the view that spatial measurement depends upon mental constructions.

I would go farther and suggest that the supposed intrinsic difference between the nature of spatial magnitudes and the nature of what I will call psychic magnitudes is illusory. It is commonly believed that space is in itself of such a nature that it can be precisely measured, while psychic elements are in themselves of such a nature that exact measurement is not applicable to them. And the marked difference between physics as an exact quantitative science and psychology as a merely qualitative science is thought to be due to the essential difference of subject-matter.

I believe that this is incorrect, and that as regards their original perceptual elements physics and psychology are on exactly the same footing. Perceived differences of size such as are involved in the comparison of the house and the man are in themselves, I suggest, differences of *degree* which are incapable of precise estimation, and are in all respects similar to perceived differences in the intensity of a felt pain or anger. The difference between physics and psychology does not reside in their subject-matter, but in the fact that in the former case the human mind has successfully invented fictitious concepts of equality and measurement which it *imposes* upon space but does not find there; while in the latter case the mind has failed to discover a suitable construction. Mathematics is no more applicable to the raw material of perceptual space than it is to pleasures and pains. It has been made applicable by the cunning devices of the human mind.

Or we may put the same thing in another way. You may say, if you like, that the nature of space itself is such that it lends itself to exact measurement, and that this is not true of psychic magnitudes. But if so, we must add that this essential nature of space is itself a mental construction.

The truth of these suggestions, which are, I suppose, rather novel, and therefore not likely at once to find a ready

entrance into the reader's mind, can only be proved in one way, namely by showing that space as perceived is *not* capable of exact measurement. This again can only be proved by showing that the equality of two spatial quantities can neither be perceived nor inferred from anything that is perceived, or in other words that it must be a mental construction. When once this is proved, it will follow that, though greater and less exist in perceptual space, this greater and less cannot be measured, without the use of fictions, any more than psychic elements can be. With this the essential difference between extensive and intensive magnitudes disappears. Let us proceed, then, to the proof that equality and measurement are constructions.

Suppose that the rigid rod AB is considered equal in length to the rigid rod CD . What is *meant* by the concept of equality here, and how is it arrived at? It appears to mean that if I pick up AB and superimpose it upon CD , I shall perceive that the two ends coincide, A with C , and B with D . If this happens the rods are called equal. This is the only meaning which can be assigned to the concept of equality.

Now the actual coincidence of two or more lengths may be given in perception. I mean that we may actually see two rods lying along each other with ends coinciding. If we choose to name this *perceived* relation 'equality', then such equality would be simply a concept of the given. It would be a perceived fact, and as such would have of course no element of construction about it.

I pass over the difficulty that, if two lines actually coincide, they become one single line, so that the relation of equality cannot with any degree of accuracy be said to hold between them. When two rods are seen to lie together in the manner described, they are actually at a very small distance from one another. So that even this concept of perceived equality cannot be given any precise meaning. But it is more pertinent to our inquiry to waive this, and to point out that this concept of perceived or given equality is not what is meant by the geometer when he speaks of equal lines or volumes, and that it is useless for any pur-

pose of measurement, and therefore for any scientific purpose. The only concept which is valuable for purposes of measurement is that which conceives of two quantities as equal *when they are apart from each other*, i.e. at a distance from one another.

When we measure a rod and say that it is six feet long, the six equal divisions of the rod do not coincide but lie apart from one another in different, though continuous, parts of space. When I say that anything is a yard long in the room in which I am writing, I mean that it is equal in length to the standard measure which may be deposited in a place several thousands of miles away. Thus the only concept of equality which is of any real service either in science or in the common measuring operations of life involves the belief that spatial magnitudes may be equal to one another when they are separated in space. Now it is this kind of equality which can neither be perceived nor proved.

That it cannot be perceived will, I think, be readily granted. I obviously cannot perceive the equality of two straight lines which are so far apart as not to lie in the same field of vision. And it is equally true that I cannot perceive their equality when they lie close together in the same field of vision. By glancing from one to the other I can perceive that they are roughly similar. But in the first place such a judgement is obviously very rough and ready. In the second place, it is merely a crude example of the method of proving equality by superimposition. For it consists in so disposing the eye that the images of AB and CD fall successively upon the same portion of the retina. Thus it is constituted by the superimposition of one retinal image upon the place just occupied by the last. But proof of equality by superimposition is entirely fallacious.

Suppose we wish to prove that $AB = CD$. They are some inches distant from one another. We pick up AB , move it a few inches, and place it upon CD . Finding that the ends coincide we pronounce the two straight lines equal. Now this process no doubt proves that when the

two lines are together they are equal, if by equality we mean actual perceived coincidence. It proves the identical proposition that when they coincide they coincide. But this is all it proves. It entirely fails to prove that the two lines are equal when they are at a distance from one another, which is the only thing that we want proved.

If the reader supposes that we are making the point that the proof is incomplete because the length of the lines might have changed during the process of superimposition, he has only gathered half the real issue. It is true that it may be urged that proof by superimposition assumes without any justification that the length of the lines remains 'the same' throughout the operation. And this assumption vitiates the proof. But the real difficulty goes much deeper than this and relates to the question what is *meant* by 'the same', or by equality, in these conditions. To say that AB remains the 'same' length for one minute means that AB *now* is equal to AB *a minute ago*. Here we get the concept of the equality of quantities separated by periods of *time*. To say that $AB = CD$ *now*, is to assert the concept of the equalities of quantities separated by distances of *space*. What do these concepts mean?

They clearly have to be interpreted in terms of actual coincidence, in terms of that kind of 'equality' which is perceived when we see that two rods are lying one on the other and that their ends coincide. To say that the two rods AB and CD , which are six inches apart, are equal, can only mean that *if* they were now lying together they would coincide. But we recognize this 'if' at once as an old friend. It imports here the impossible condition in the antecedent of the hypothetical proposition which expresses the existential construction. It is impossible that while AB and CD are six inches apart they should at the same time be coincident. Yet this is the meaning involved in the idea of equality. For the proposition $AB = CD$ does not assert a relation between them *in the future* (when they are brought together), but *now* (when they are apart). The *present* equality of AB and CD , when they are apart, can never be proved, because when they are brought together

the present is gone, and their coincidence a minute hence does not prove anything about their relations *now*. Hence the equality of quantities separated in space can never be proved.

It does not help to reply that in order to prove that $AB = CD$ we can take a third rod EF , place it along AB till the ends coincide, carry it across the intervening distance to CD , make it coincide with CD , and thus prove the equality of AB to CD . For this proves nothing of the kind. It proves that $AB = EF$ at the moment when they coincide, and that $EF = CD$ at some later moment when they coincide. But in order to prove that $AB = CD$ it has to be shown that EF , during the moment when it is coinciding with CD , is equal to AB , or in other words that $AB = EF$ when they are separated by a distance of space. Our proof assumes that this is true. But this is itself the very principle we set out to prove, namely that two rods may be equal when separated by a distance. The argument is a *petitio principii*. Thus it is impossible to prove by any method whatever, that two lengths of space are equal to one another. And the same considerations of course apply to areas and volumes.

It is hardly sufficient to say that the concept of equality at a distance is an assumption. For to say that the proposition $AB = CD$ is assumed to be true implies that although we cannot prove it to be true we can at least give it a clear meaning. But our real difficulty with the concept of equality is that we cannot attach to it a clear and consistent meaning. We understand what is meant by the proposition $AB = CD$ when the two are actually coinciding. The equality is then merely another name for the perceived fact of coincidence. But when they are not coinciding, what does it mean? It can only mean that *if* they were brought together they *would* be equal, i.e. coincide. It thus asserts something about a possible future. But it does not assert any intelligible relation between the two straight lines *now*. The concept of equality purports to express a relation which actually now exists between two things, but it actually expresses only a relation which they would have

if . . . In other words it does not express any *existent fact* at all.

It will be recognized, I think, that all this is merely another way of saying that equality is an existential construction. It possesses all the special marks of such a construction. For (1) it asserts the existence of a relation which is incapable of proof or verification. (2) It can only be expressed in an hypothetical proposition with an antecedent which states an impossible condition. (3) It is an extension of a concept of the given into the void where nothing is or can be given. The concept of equality, meaning actual coincidence, is applied by the mind to cases where there is no coincidence, just as the mind applied the concept of *percipi*, i.e. existence, where nothing is perceived. And finally (4) the mind's construction is composed of materials taken from the given. The material in this case is actual coincidence which is given in perception.

This result should serve as a useful commentary on the theory of the external world as a mental construction which was set forth in Chapter VI. If the reader is 'tough-minded' he will very likely have thought that theory fantastic. That the solid world is created by individual minds out of their private phantasms; that the table is not factually there when no one is looking, but is a construction of the imagination; these views, it may be thought, are merely intellectual curiosities. For it is obvious in spite of all sophistries that the table does exist when no one is looking at it.

But I would have the 'tough-minded' reader reflect upon the parallel case of the concept of spatial equality. Our views upon external existence will no doubt be disputed by some competent philosophers. But that it is impossible to prove equality of lengths or other magnitudes, or even to attach a clear and direct meaning to the conception, will be admitted, I think, by every competent physicist and mathematician. This leads direct to the view that the concept of equality, and therefore the concept of measurement, are mental constructions having

exactly the same characters as those which we have attributed to the construct of external existence. But tough-mindedness and common sense, if left to themselves, would unhesitatingly condemn such views as far-fetched and fantastic nonsense. Which shows that we cannot trust common sense and tough-mindedness. It is better to trust in reason carried to its final conclusions. Common sense is ever ready to follow reason so long as its conclusions are familiar and expected. But as soon as a strange and unusual conclusion, calculated to shock the average unintelligent mind, is reached, common sense deserts reason and turns against it. Common sense would certainly have dismissed as fantastic the physical theories of Einstein and Niels Bohr. But science has become too strong for its views to be ridiculed out of existence by common sense. Philosophy unfortunately is not in this strong position. Yet the tough-minded reader who feels inclined to dismiss as nonsense the theory of independent external existence as a mental construction might do well to reflect on these considerations.

It will have to be admitted by every competent judge that in such matters as the measurement of spatial magnitudes the mind constructs its concepts, and that they cannot be found in the given. Is it not antecedently probable that if we dig back into the obscure beginnings of the mind's fundamental ideas of existence, independent externality, and the like, we shall find similar constructional operations being carried out? Is it not probable that knowledge is all of a piece, in its underground foundations as in its superstructure?

Eighth Construction.

That there exist relations of equality between periods of time; and that the measurement of time becomes thereby possible.

Here the constructive character of our concepts is more obvious than it was in the case of space. For it was pointed out long ago, by John Locke in fact, that whereas we can superimpose spatial lengths upon one another, it is impossible to perform a similar operation for time periods.

We can take the rod AB and place it upon the rod CD . But we cannot take the present period of sixty seconds and place it upon a preceding or succeeding period of sixty seconds, and so 'prove' the two periods equal.

Superimposition as a proof of spatial equality is fallacious for the reasons discussed. But even this fallacious proof fails us in the case of time. It cannot be applied. Time periods are measured by clocks, whether these clocks consist of pendula, atoms, the earth moving in its orbit, or other such. But nothing can prove that two swings of a pendulum, two complete orbital movements of an electron, or of the earth, occupy *equal* times.

Spatial equality defined as actual coincidence is *given*. But there is no kind of temporal equality which is given. Therefore to find a meaning for temporal equality we have to go back to the given kind of spatial equality, and interpret it in terms of that. In other words equality of time is thought of on the analogy of equality of space, and is based upon an artificial application of spatial concepts to time.

To assert the equality of two periods of time can only mean 'If it were possible to superimpose one period on the other, in the same way in which it is possible to superimpose one spatial length on another, they would coincide'. One could not find a case in which it is more obvious that the antecedent of the proposition which expresses the construction states an impossible condition.

It is also obvious that the concept of temporal equality is a further extension of the idea of coincidence (which is a concept of the given) into a void where that idea has no existence and no real applicability. There is a double extension involved. The concept of actual space-coincidence is first extended to spaces which are not coincident. This gives us the construction of the equality of spaces at a distance from one another. This idea is then further extended to time.

It is obvious finally that the concept of temporal equality is constructed ultimately out of the material of the given, i.e. out of the actual coincidence of objects in space.

Thus it possesses all the characteristics of the existential

construction. The existence constructed is, of course, a relation, not a 'thing'.

We began with the many private extension-spreads and duration-spreads of the many solitary minds. By means of the eight constructions detailed in this chapter we have advanced to a single continuous public space and a single continuous public time. Visual space which began with two dimensions has been seen to develop three. Empty space has made its appearance. Tactile space has been identified with visual space. Space and time have both become measurable. In a word we have advanced from the rudimentary position of the solitary mind, with its many private perceptual spreads, to the space and time of common everyday knowledge. We have advanced to the space and time which ordinary men conceive as surrounding and containing the universe in which they live.

But the conceptions of equality and measurement which came last in the process of development bring us to the threshold of the more advanced questions of geometry and physics. It is through the seventh and eighth constructions alone that these sciences become possible. These constructions are their foundations. These more advanced questions belong, however, not to common everyday pre-scientific knowledge, but to science itself. And since the aim of this chapter is to proceed only as far as the position of common knowledge, I shall stop here. Questions of Euclidean and non-Euclidean spaces and space-times will be treated under the head of advanced knowledge in Chapters XI and XIV. And I shall have some few observations to offer on the subject of the space-time of relativity mechanics under the head of scientific knowledge in Chapter XIV.

There are, however, two other topics to which I will here briefly refer before closing this chapter. They are (1) the question of the infinity or finitude of space, and (2) the question of fourth, fifth, and further dimensions of space. It is true that neither of these topics belongs to the realm of common knowledge. Both are advanced and, in

their scientific aspect, abstruse. But their epistemological aspect, which is all that concerns us, can be quite simply treated. And it will be convenient to make here what brief remarks I have to offer on these subjects.

The infinity of space has been accepted in the scientific world until very recently. Now, under the influence chiefly of relativity mechanics, astronomers and physicists have begun to posit various kinds of finite space. Epistemology is not concerned to decide which view is true. That is a question for the special sciences. Our only interest in the matter is to note that both views alike are constructions. The present issue in the scientific world may be defined as the question which of the two (or more) constructions is true.

The way in which the construction of infinite space has developed is fairly obvious. It depends on the prior construction of empty space. When once empty space has been invented, when once space is believed to extend beyond the edges of objects, there appears to be no reason for supposing that it stops anywhere. Infinite space is thus an inference, not from any perceived fact (in which case it would not be a construction, but a fact) but from previous constructions, especially from that of empty space. Its own existence is therefore constructive.

That the concept of finite space is also a construction will be evident for the same reasons. For a finite space, too, unless it is confined to actual perceived spaces, involves the idea of empty space. Moreover it depends upon other constructions as well. These constructions, which belong to the current body of scientific thought, are more or less abstruse in their nature. But it will suffice for our purpose to point out that finite space is a deduction from certain geometrical 'axioms'. I shall show in the chapter on mathematical knowledge that those axioms which are not analytical propositions are mental constructions. And since finite space is a deduction from those axioms which are constructions, it must be itself a construction.

Which of the rival constructions is 'true' is, as already stated, a question for the scientists. What in general dis-

tinguishes a valid or true construction from an invalid one (i.e. from a mere figment of the imagination) is a question for epistemology. But it is a question which cannot be answered until our investigations into the nature of knowledge are concluded. We shall have to face it in a later chapter.

A word, finally, about modern speculations regarding possible four-dimensional, five-dimensional, and n -dimensional spaces. The question is *not*, it must be carefully noted, anything to do with the four-dimensional *space-time* of relativity physics. For the fourth dimension in that case is time. It is not a fourth dimension of space. To confuse these two ideas is a common mistake in the popular mind, and one which we must, of course, avoid. The question which we are here considering is that of the possibility of genuine *spatial* dimensions beyond the third.

The attitude of epistemology to the problem is quite simply stated. It is this. The human mind has already constructed the third visual dimension, and has consolidated its position around it. It has become embodied in the everyday knowledge of the mind. It is no longer strange, but is taken as a matter of course. This construction was made necessary in order that the mind might explain its experiences (chiefly the experience of change) consistently with the concept of an independent public external existence. There is not the slightest reason why the mind should not construct any number of further dimensions if they too are found necessary to bring consistency into its knowledge, as that knowledge advances. It is true, of course, that such further dimensions will be inventions. But so is the third visual dimension. And so are the greater number of 'truths' which constitute knowledge. The fourth and further dimensions will be just as 'true' and as 'real' as the third dimension, or indeed as the independent existence of the external world, which is surely the most firmly established of 'truths'.

Such a necessity for constructing further dimensions is now, apparently, beginning to be felt in the physics and

astronomy of to-day. And there is nothing impossible in the prediction that the men of some far future date may come to take these further dimensions for granted in exactly the same way as we now take the third dimension. And then a further strange development may take place. The human mind, having long ago constructed the third dimension, *forgot* the processes of construction. They sank into the unconscious. Hence we now *read* the third dimension into perception. We imagine that we *see* distance and depth. Just in the same way the men of the far future, having constructed an n -dimension, may forget the constructions. All the intellectual processes connected with them may become buried in the unconscious. Then those men will imagine that they actually perceive the n -dimension, see it with their eyes, feel it with their hands. And if so, we may expect that the realist philosophers of that time will be misled. They will argue that, because the fourth and fifth dimensions are actually experienced as seen, they are therefore part of the visual given, and that belief in their existence as independent of mind is a 'primitive belief'. And any one who says that they were once constructed, and are not 'there' independently of such construction, will be derided in the name of common sense.

There is one consideration, however, which may give pause to these pleasant prophecies. The construction of the third visual dimension took place in the remote past, presumably when man was emerging from the brute, or perhaps earlier than that. It was therefore always unconscious, whereas the present constructions of science are being consciously carried out and recorded. Perhaps for this reason they will not be forgotten.